

CLAREMONT McKENNA COLLEGE

TREASURY BOND OPTIONS AS A HEDGE  
AGAINST FORWARD MORTGAGE COMMITMENTS

SUBMITTED TO  
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## CHAPTER I

### INTRODUCTION: OVERVIEW

Interest rate movements have a tremendous impact on this country's economic climate. Corporations, small firms, and individuals react to and are affected by fluctuating interest rates, even if they are not intimately involved with the capital market. Over the past decade this fact has become increasingly clear to investors and managers as interest rate risk has materialized. To analyze investments effectively, a manager/investor must take into consideration current interest rate yield curves and the probability of significant changes in the future. Extreme interest rate movements contribute to foregone interest income, decreased asset values, increased liability costs, and increased exposure during loan commitment periods. Often managers or individuals must postpone borrowing until capital needs are acute, and then they must determine whether to pay high interest costs or not borrow at all. Hence, possible interest rate fluctuations increase the risk inherent in any investment.

Since interest rate risk can have an adverse effect on what otherwise would be prudent investments, this paper will examine an interest rate management tool which may be applicable in reducing or eliminating interest rate risk.

More specifically, the utility of United States Treasury Bond options as a hedge against forward mortgage commitments will be analyzed.

The rest of this chapter describes and provides examples of two key concepts used later in the analysis, and introduces and describes what mortgages are, what mortgage bankers do, and what forward commitments are and how they can be used to decrease a borrower's interest rate risk. Chapter two discusses what interest rate options are, introduces the T-bond option contract, and provides an analysis of a model that seeks to determine the fair value of an option. Chapter three illustrates the use of T-bond options as a tool for managing interest rate risk with hypothetical hedging examples.

### Interest Rate Risk

Interest rate risk can be incurred under a wide array of circumstances, and can be generally defined as a probable, detrimental result of a future change in interest rates.<sup>1</sup> To illustrate, consider the following scenario of purchasing an apartment. On May 13th an investor decides to purchase an apartment complex with a purchase price of five million (\$5,000,000) dollars. The investor has \$500,000 for a down payment, and applies for a 30 year, fixed-rate mortgage. Currently mortgage interest rates for this type of loan are quoted by lenders at 12 percent. Using this

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<sup>1</sup>Allan M. Losigian, Interest Rate Futures (Princeton: Don Jones Books, 1980), p. 156.

rate as a basis for deciding to purchase the apartment, the investor determined, with 12 percent as an annual interest charge, that monthly mortgage payments would equal \$46,287.57 using the amortization schedule in Exhibit 1-A.

However, sixty days later when the loan application was approved and certain agreements made to get the mortgage through escrow, the investor ended up with an annual percentage rate (APR) on the loan of 14 percent. During the period between application for the loan and its closing date, mortgage rates increased 2 percent from 12 percent to 14 percent. The apartment owner now has to make monthly mortgage payments of \$53,319.23 based on the amortization schedule in Exhibit 1-B.

The difference between what the investor originally expected to pay per month and what he now pays equals \$7,031.66 ( $\$53,319.23 - \$46,287.57 = 7,031.66$ .) In nominal terms, the rate increase has made the cost of the apartment complex increase by \$2,531,397.60 ( $\$7,031.66 \times 12 \text{ months} \times 30 \text{ years}$ .) By assuming that the apartment owner could have invested the monthly difference in an account earning 12 percent simple interest compounded monthly, the present value of the difference in payment amount, discounted at 12 percent, equals \$683,606.25. Based on the following formula

EXHIBIT 1-A

AMORTIZATION SCHEDULE:

MORTGAGE PRINCIPAL= \$4,500,000.00  
 AMORTIZATION PERIOD: 30 YEARS  
 NOMINAL INTEREST RATE: 12.00%

MONTH	BEGINNING LOAN BALANCE	MONTHLY PAYMENT	INTEREST PORTION	PRINCIPAL REDUCTION	ENDING LOAN BALANCE
1	\$4,500,000.00	\$46,287.57	\$45,000.00	\$1,287.57	\$4,498,712.43
2	\$4,498,712.43	\$46,287.57	\$44,987.12	\$1,300.44	\$4,497,411.99
3	\$4,497,411.99	\$46,287.57	\$44,974.12	\$1,313.45	\$4,496,098.54
4	\$4,496,098.54	\$46,287.57	\$44,960.99	\$1,326.58	\$4,494,771.96
5	\$4,494,771.96	\$46,287.57	\$44,947.72	\$1,339.85	\$4,493,432.11
6	\$4,493,432.11	\$46,287.57	\$44,934.32	\$1,353.25	\$4,492,078.87
7	\$4,492,078.87	\$46,287.57	\$44,920.79	\$1,366.78	\$4,490,712.09
8	\$4,490,712.09	\$46,287.57	\$44,907.12	\$1,380.45	\$4,489,331.65
9	\$4,489,331.65	\$46,287.57	\$44,893.32	\$1,394.25	\$4,487,937.39
10	\$4,487,937.39	\$46,287.57	\$44,879.37	\$1,408.19	\$4,486,529.20
11	\$4,486,529.20	\$46,287.57	\$44,865.29	\$1,422.27	\$4,485,106.93
12	\$4,485,106.93	\$46,287.57	\$44,851.07	\$1,436.50	\$4,483,670.43
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358	\$138,862.71	\$46,287.57	\$1,388.63	\$44,898.94	\$92,575.13
359	\$92,575.13	\$46,287.57	\$925.75	\$45,361.82	\$46,287.57
360	\$46,287.57	\$46,287.57	\$462.88	\$45,824.69	\$0.00

EXHIBIT 1-B

AMORTIZATION SCHEDULE:

MORTGAGE PRINCIPAL= \$4,500,000.00  
 AMORTIZATION PERIOD: 30 YEARS  
 NOMINAL INTEREST RATE: 14.00%

MONTH	BEGINNING LOAN BALANCE	MONTHLY PAYMENT	INTEREST PORTION	PRINCIPAL REDUCTION	ENDING LOAN BALANCE
1	\$4,500,000.00	\$53,319.23	\$52,500.00	\$819.23	\$4,499,180.77
2	\$4,499,180.77	\$53,319.23	\$52,490.44	\$828.79	\$4,498,351.98
3	\$4,498,351.98	\$53,319.23	\$52,480.77	\$838.46	\$4,497,513.53
4	\$4,497,513.53	\$53,319.23	\$52,470.99	\$848.24	\$4,496,665.29
5	\$4,496,665.29	\$53,319.23	\$52,461.10	\$858.13	\$4,495,807.16
6	\$4,495,807.16	\$53,319.23	\$52,451.08	\$868.15	\$4,494,939.01
7	\$4,494,939.01	\$53,319.23	\$52,440.96	\$878.27	\$4,494,060.74
8	\$4,494,060.74	\$53,319.23	\$52,430.71	\$888.52	\$4,493,172.22
9	\$4,493,172.22	\$53,319.23	\$52,420.34	\$898.89	\$4,492,273.33
10	\$4,492,273.33	\$53,319.23	\$52,409.86	\$909.37	\$4,491,363.96
11	\$4,491,363.96	\$53,319.23	\$52,399.25	\$919.98	\$4,490,443.98
12	\$4,490,443.98	\$53,319.23	\$52,388.51	\$930.72	\$4,489,513.26
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358	\$159,957.69	\$53,319.23	\$1,866.17	\$51,453.06	\$106,638.46
359	\$106,638.46	\$53,319.23	\$1,244.12	\$52,075.11	\$53,319.23
360	\$53,319.23	\$53,319.23	\$622.06	\$52,697.17	\$0.00



for computing the present value of an annuity:<sup>2</sup>

$$PV_a = d \frac{(1 + r/m)^{tm} - 1}{(r/m)(1 + r/m)^{tm}} ;$$

where  $d = 7,031.66$  (monthly difference)

$r = .12$  (discount rate)

$t = 30$  (term of loan)

$m = 12$  (compounding term)

In the above example the investor experiences, over the life of the mortgage, the adverse effect of a short term interest rate increase. This would be especially true if there were a prepayment clause which stated that upon early prepayment a 5 percent penalty would be charged because if mortgage rates fell in the future, it would be more costly to refinance. This example illustrates the result of an actual rate increase; whereas, interest rate risk entails the possibility that interest rates will change adversely over a critical period of time. The critical period in this example was sixty days. Interest rate risk in this case can be stated as the possibility that mortgage rates will increase before closing.

Interest rate risk can also be incurred as the result of a decrease in interest rates. For example, if an investor chooses to deposit an amount in a variable rate account currently yielding 10 percent annually over a fixed rate account with an expiration of one (1) year and an

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<sup>2</sup>Ronald K. Teeple, "Value Concepts: Present Calculations -- Future Quantities," notes for Economics 176 at Claremont McKenna College.

annual rate of 9.5 percent, there exists a potential risk if interest rates fall. If rates decrease, the variable account's yield will decrease while the fixed account's yield can not. Even if the funds deposited in the variable account were withdrawn, a yield with an equivalent risk and yield to the one year, fixed-rate account's probably could not be found. Hence, interest rate risk can also be incurred from falling rates.

### Hedging

Hedging can be defined as a method of protecting an established or committed position by a counterbalancing transaction.<sup>3</sup> A hedged position can be developed so that different degrees of risk or no risk is involved.<sup>4</sup> A position that has no risk involved is considered a perfectly hedged position. A perfectly hedged position dictates that all possible outcomes from a position result in a predetermined yield without variance. While a perfectly hedged position is theoretically possible, in reality, due to the unpredictability of market prices, demand and supply, a perfect hedge is unattainable.<sup>5</sup> That is, a perfect hedge is unattainable because the market for one side of a transaction is never perfectly matched with the other. In order for both sides of a market to be

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<sup>3</sup>Loosigian, p. 316.

<sup>4</sup>Ibid., pp. 315-349.

<sup>5</sup>Ibid., p. 323.

perfectly matched, prices must move in exactly opposite directions, at the same time by the same amount. Hence, only when two markets have an absolute correlation of  $-1.0$  can a perfect hedge be set up. However, by determining the correlation between the established position's market and the corresponding counterbalancing transactions market, and adjusting on the side of the counterbalancing transaction to correct for a risk, a hedge that is almost perfect can be set up. The following is an adapted example of an imperfect hedge using GNMA futures to hedge the issuance of mortgage-backed bonds.<sup>6</sup>

#### Situation

The management team of a local savings and loan association recently concluded an analysis of alternative sources of long-term funds. In January of 19XX the analysis indicates that the issuance of a mortgage-backed security (Debit collateralized by a pool of mortgage loans) would provide the lowest long-term financing.

On February 1, 19XX, an investment banking firm informs the management team that they can issue debt at 12.75 percent.

The management team predicts that it would take at least three months to close the funds in residential mortgages. During the interim period (the time before the

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<sup>6</sup>Managing Interest Rate Risk with Financial Futures (St. Louis: Clayton Brokerage Company, [1982]), pp. 24-5.

funds from the debt issuance are invested in mortgages) the management team plans to invest the funds in a 90-day Certificate of Deposit (CD) with an annual rate of 14.5 percent. Current mortgage rates are 13.5 percent made by the savings and loan, hence, the net interest spread equals 75 basis points ( $13.5 - 12.75 = 75$ ), one basis point equals 1/100th of a percent. The management team insists on a basis yield of at least 60 points in other like deals. Hence, they perceive the situation as satisfactory as long as the basis margin remains above 60 points. The management team notes that if mortgage rates decline over the three month interim period, the basis margin will decline. On the other hand, if mortgage rates increase during the interim period, the basis margin will improve.

To stabilize the basis margin at 75 points, the management team determines that a hedging transaction using a GNMA futures contract should be employed.

#### Major Assumptions

1. The savings and loan issues \$10,000,000 in mortgage-backed bonds on February 1, 19XX, at a rate of 12.75 percent
2. The funds are invested in a 90-day CD with a rate of 14.5 percent
3. The funds from the bond issuance are used to fund \$10,000,000 in residential mortgages on May 1, 1984 (it would be more realistic to assume incremental funding, but for simplicity, the above assumption will hold.)

### Hedge Structure

The hedge structure will focus on the net interest margin between the 12.75 percent long term debt and the future yield on the mortgages. Since the yield on the mortgages will not be known until May (the margin will be at risk until then) long futures positions are needed to make the margin remain constant. The futures contract employed to hedge the spread will be a GNMA futures contract because GNMA yields and mortgage yields have a high correlation. Hence, the savings and loan purchases a long position in a June 19XX contract.

The number of GNMA contracts needed to hedge the future mortgage issuance is determined by the following formula:

$$\frac{\text{Portfolio par}}{\text{Contract par}} \times \text{Conversion Factor} = \begin{array}{c} \text{Number} \\ \text{of} \\ \text{Contracts} \end{array}$$

$$10,000,000/10,000 \times 1.414 = 141.1 \text{ contracts;}$$

where Conversion Factor = Price Value per Basis Point of the Security to be Hedged/Price Value per Basis Point of Hedging Vehicle.

The conversion factor 1.414 is employed because it is the reciprocal of the published factor for the current 13.5 percent GNMA mortgage certificates. This conversion factor is determined by changing the yield on both securities by .01 percent and solving for their price or present value. Since 141.4 is not a whole number (contract amounts must be in increments of \$100,000) 141 contracts are bought.

### Conversion Factors

The conversion factor seeks to equalize a hedged position's volatility from the difference in the dollar value of security prices from a change of one basis point between the instrument to be hedged and the hedging vehicle. The conversion factor is defined by:<sup>7</sup>

$\frac{\text{Cash B.P.}}{\text{Option B.P.}}$  ; where

Cash B.P. = The dollar value of a change of one basis point in the hedged security.

Option B.P. = The dollar value of a change of one basis point for the hedging medium to be employed.

### Results

Assuming that mortgage rates fell 200 basis points from 13.5 percent on February 1, 19XX, to 11.5 percent on May 1, 19XX, the net interest margin of 75 points decreased by 200, to -125 basis points. On the other hand, during the same time period, the 13.5 percent GNMA futures contract's price rose 275/32nds. This resulted in total contract gains of \$1,211,190 calculated as follows:

Number of Contracts (Par Value) [GNMA Basis Increase]/<sup>100</sup>

$$141(100,000[(275/32)/100] = 1,211,718.75$$

The book value of the mortgages (10,000,000) is adjusted by the amount of the futures contract gains

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<sup>7</sup>Ibid., p. 3.

(\$-1,211,718.75) to arrive at an adjusted book value of \$8,788,281.25. This adjustment of the cost basis effectively increased the yield on the mortgages from 11.50 to 13.57 percent (assuming that all servicing fees and points are excluded), by incrementally accruing the futures gain plus its associated earnings into mortgage loan income. The GNMA futures contracts were closed out at the time of mortgage issuance; i.e., on May 1, 19XX.

The results of the hedging example are illustrated in Exhibit 1-C. If the management team had decided not to hedge the mortgages, a loss of \$10,416.67 would have been incurred monthly on the negative net interest margin until the mortgages were sold at a loss, or until the long term debt was repurchased at a substantial cost. Using GNMA futures to hedge the mortgages resulted in a positive \$5,166.67 per month throughout the maturity of the mortgage loans and/or the mortgage backed debt. The difference in net monthly returns from the two positions equals \$15,583.33.

Exhibit 1-D summarizes the result from the hedged and unhedged positions. It must be pointed out that the hedge was not perfect because the net interest margin declined to 62 points from 75 basis points. However, 93.5 percent protection should be considered better than being at risk 100 percent. The 6.5 percent short fall was caused by the fact that the yields on mortgages decreased faster than did yields on GNMA futures.

EXHIBIT 1-C  
 FUTURES HEDGING EXAMPLE  
 UNHEGED RESULTS:

DATE:	DEBT EXPENSE	CD INCOME	MORTGAGE INCOME	NET INCOME
FEBRUARY '84	\$106,250.00	\$122,916.67	\$0.00	\$16,666.67
MARCH '84	\$106,250.00	\$122,916.67	\$0.00	\$16,666.67
APRIL '84	\$106,250.00	\$122,916.67	\$0.00	\$16,666.67
MAY '84	\$106,250.00	\$0.00	\$95,833.33	(\$10,416.67)
TOTAL:	\$425,000.00	\$368,750.00	\$95,833.33	\$39,583.33

HEDGED RESULTS:

DATE:	DEBT EXPENSE	CD INCOME	MORTGAGE INCOME	NET INCOME
FEBRUARY '84	\$106,250.00	\$122,916.67	\$0.00	\$16,666.67
MARCH '84	\$106,250.00	\$122,916.67	\$0.00	\$16,666.67
APRIL '84	\$106,250.00	\$122,916.67	\$0.00	\$16,666.67
MAY '84	\$106,250.00	\$0.00	\$111,416.67	\$5,166.67
TOTAL:	\$425,000.00	\$368,750.00	\$111,416.67	\$55,166.67

(1) DIFFERENCE = \$15,583.33

(1) The difference of 15,583.33 would be repeated for each month throughout the maturity of the mortgage loans and/or the mortgage backed debt.

"Managing Interest Rate Risk with Financial Futures", Claton Brokerage Corp., St. Louis, MO (1982) p.25.



EXHIBIT 1-D  
FUTURES HEDGING EXAMPLE  
SUMMARY

UNHEDGED RESULTS:

^^  
BOOK VALUE OF MORTGAGES                      \$10,000,000.00  
YIELD ON MORTGAGES    11.5  
RATE PAID ON DEBT    12.75  
NET INTEREST SPREAD    -125

HEDGED RESULTS:

^^  
BOOK VALUE OF MORTGAGES                      \$8,788,280.00  
YIELD ON MORTGAGES    13.77  
RATE PAID ON MORTGAGES    12.75  
NET INTEREST SPREAD    62

In the above hedging example, the savings and loan faced interest rate risk on the future issuance of mortgages. To eliminate or decrease this risk, the management team entered into counterbalancing transactions that effectively hedged their position. Although the hedge was not perfect, the use of a counterbalancing transaction turned out much better on a profitability basis than what would have otherwise occurred without hedging. Hence, the practicality of engaging in hedging operations can be seen.

### Mortgages

Real estate mortgage loans are the most common method of financing real estate acquisitions.<sup>8</sup> A mortgage loan requires the borrower (the mortgagor) to pledge the acquired real property as security for the loan.<sup>9</sup> Under a mortgage arrangement, the terms of borrowing are agreed upon by the mortgagor and the mortgagee (the Lender) and the note is signed by both parties. A more formal definition of a mortgage states:

A real estate mortgage is a lien on an interest in the land, created by a formal agreement, by a transfer of such interest, to service payment of money of the performance of some other act.<sup>10</sup>

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<sup>8</sup>William B. Brueggman and Leo D. Stone, Real Estate Finance, 7th ed., (Homewood, Ill: Richard D. Irwin, Inc., 1981), p. 5.

<sup>9</sup>Ibid.

<sup>10</sup>H.T. Tiffany, A Treatise on the Modern Law of Real Property (Chicago: Callaghan & Co., 1939), p. 912, quoted Ibid., p. 6.

All mortgages must have the following contents: identification of mortgagor and mortgagee; "covenants of seign and warranty", legal description of the real property, and "provisions for release of power by the mortgagor."<sup>11</sup> While these elements are requirements for any type of mortgage, there are many other important clauses usually incorporated in a mortgage note. A few of these clauses are identified in Appendix A.

Mortgage loans may have one of a growing number of repayment patterns. The difference between a term loan and an amortized loan is that under a term loan only interest is payable during the life/term of the loan; while an amortized loan involves periodic payments (usually monthly) including both interest and principal. The most common form of mortgage loan is the straight-term mortgage.<sup>12</sup> There are four ingredients which determine a straight-term mortgage loan's repayment pattern, they are: the nominal interest rate, the principal amount, the amortization period, and the loan maturity date. Under a straight-term mortgage, the amortization period and the period before the maturity date are equal. The following is an example of a fully amortized, constant payment mortgage loan: Assume that an individual receives a \$100,000 mortgage loan with a maturity and amortization period of 30 years with a nominal interest rate of 12 percent. By employing the

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<sup>11</sup>Ibid., p. 7.

<sup>12</sup>Ibid., p. 138.

formula<sup>13</sup>:

$$p = P \frac{(r/m)(1+r/m)^{mt}}{(1+r/m)^{mt} - 1} ;$$

where m is greater than one and finite,

P = Principal loan amount,

r = nominal interest rate,

m = # of yearly payments,

t = # of years to maturity,

p = payment amount.

The constant monthly payment equals \$1,028.61. The mortgage payment pattern for the life of this mortgage is shown in Appendix "C". Exhibit 1-E illustrates the relationship between the amount of interest and principal paid over the life of the mortgage. As the mortgage's date nears, interest paid decreases and principal reduction increases, in the exhibit, they cross in month 292.

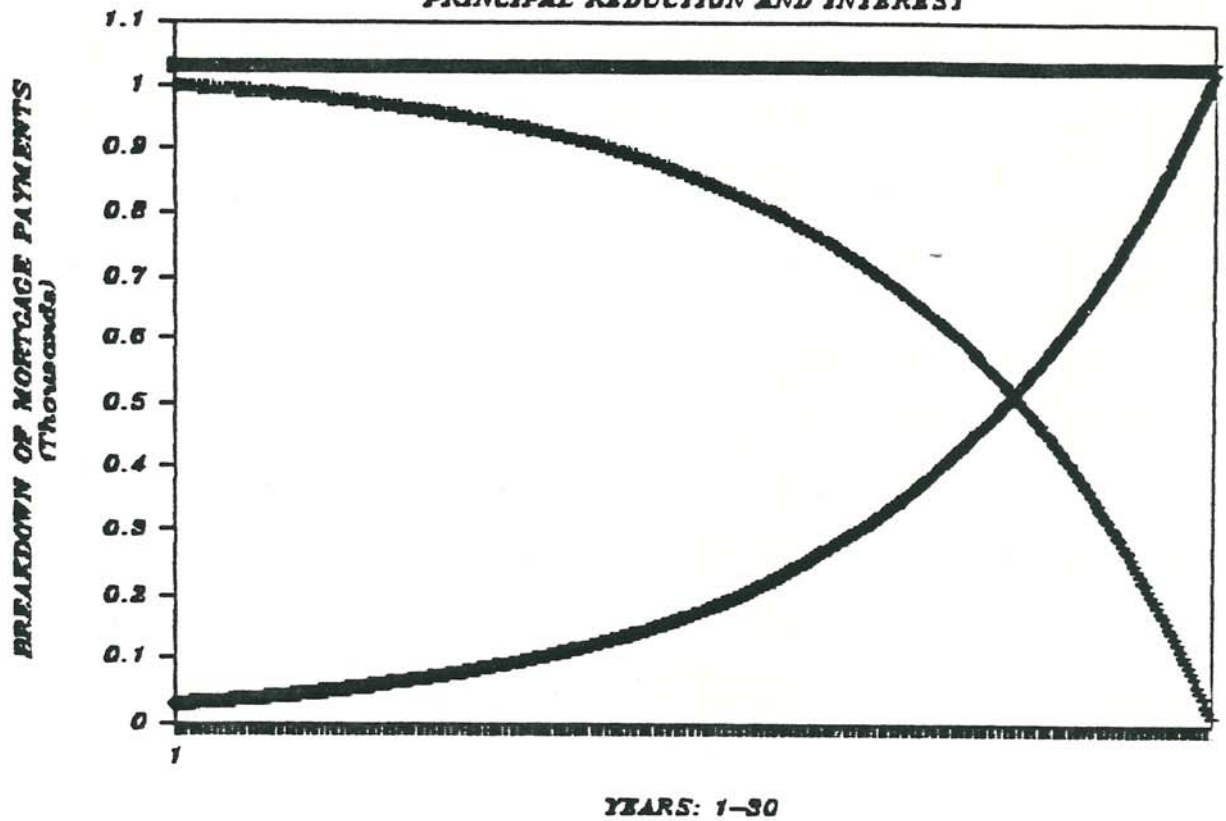
Although there are many other different forms of mortgages, with various repayment patterns, e.g., adjustable rate mortgages, graduated payment mortgages, flexible payment mortgages...etc.,<sup>14</sup> the purpose of defining and providing an example of a straight-term, fully amortized mortgage loan was to make certain, later in the analysis, that the loan characteristics are understood. The analysis below will assume straight-term, fully amortized mortgage

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<sup>13</sup>Teeples.

<sup>14</sup>Brueggman and Stone, pp. 213-246.

**EXHIBIT 1-E**  
**PRINCIPAL REDUCTION AND INTEREST**



loans as the security to be hedged.

### The Mortgage Banker

Mortgage bankers, also called mortgage companies or dealers, perform an integral function as intermediaries between institutional investors and individual borrowers.<sup>15</sup> They provide a channel through which the smooth flow of long term capital is distributed. Mortgage bankers usually do not lend their own financial resources,<sup>16</sup> they act as agents or correspondents for governmentally sponsored, insured institutions such as the Federal Housing Administration (FHA), the Veterans Administration (VA), the Government National Mortgage Association (GNMA) and other financial institutions such as banks, savings and loans, the Federal National Mortgage Association (FNMA), the Federal Home Loan Mortgage Corporation (FHLMC) and others. A formal definition of who and what mortgage bankers are is located in Appendix B.

Mortgage bankers usually employ commissioned loan representatives who work with potential home buyers and developers to arrange and gather the needed information and verification for loan approval.<sup>17</sup> After a mortgage loan has

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<sup>15</sup>Miles L. Colean, Mortgage Companies (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962), pp. 4-7.

<sup>16</sup>John P. Wiedemier, Real Estate Investment 2nd. ed. (Reston, VA: Reston Publishing Company, Inc., 1982), p. 91.

<sup>17</sup>Interview with Stephen S. Keith, Joel Packer, Doug Winn, The Hammond Company, Newport Beach., CA, February 24, 1984.

been approved, mortgage bankers temporarily fund the loan with their own resources or "lines of credit" before a "pool" of mortgages are brought together and "packaged" for delivery to an institutional lender, or sold in the secondary mortgage market.<sup>18</sup> After a loan has been sold or delivered, in accordance with or without a commitment contract, the mortgage banker provides the function of servicing the loan.<sup>19</sup> Servicing generally entails collecting monthly payments and distributing the proper amounts to escrow accounts for taxes, insurance, and payment to the institutional lender.

Mortgage bankers generate most of their revenues from origination and servicing fees on mortgages on residential and income properties.<sup>20</sup> They also may generate profits or losses from the pooling of mortgages, through temporary financing, for resale when commitments have not been made with an institutional investor. If over the pooling or "warehousing"<sup>21</sup> period interest rates on mortgages decrease, the mortgage banker makes an extra profit. Conversely, if rates increase, significant losses can be incurred on the

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<sup>18</sup>As discussed with executives of the Hammond Company (a mortgage banking firm) pools of mortgages are currently packaged into what are called Collateralized Mortgage Obligations (CMO's). They are sold to investors in the secondary market.

<sup>19</sup>Brueggman and Stone, p. 575.

<sup>20</sup>Ibid., p. 586.

<sup>21</sup>The Hammond Company, Interview.

sale of the mortgage pool. Since the present value of a mortgage is affected by interest rates in a manner similar to a bond, a mortgage banker typically will engage an investment banking firm to develop and execute a hedging plan to avoid possible losses from the sale of mortgage pools that are not committed.<sup>22</sup>

Mortgage bankers often face the problem of having to make or wanting to make, forward commitments to developers that have one-sided interest rate risk. That is, forward commitments to developers obligate the mortgage banker to take delivery of mortgages at a specified price or yield, while at the same time the delivery of the mortgages is uncertain and depends primarily on market conditions.

For example, if mortgage interest rates fall, a few or no mortgages will be closed and delivered under the commitment contract. On the other hand, if rates increase, a greater proportion of the total dollar amount of mortgages committed will be closed and delivered. Hence, forward commitments represent a one-sided risk to the mortgage banker. If mortgage rates are expected to rise over the commitment period, a fully hedged position is needed to offset losses. Or if rates are expected to fall during the commitment period, no hedge is required. However, since mortgage rates are generally unpredictable, it is considered wise to hedge a position even if rate expectations are

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<sup>22</sup>Interview with Douglas M. Hayes, A.G. Becker Paribas, Inc., Los Angeles, CA, March 8, 1984. A.G. Becker Paribas provides hedging services for Weyerhaeuser Mortgage Corp.



promising.<sup>23</sup> The reason a given position needs to be fully hedged under a falling mortgage rate scenario is that when rates fall, fewer mortgages are delivered under the commitment agreement (as noted above).

The "asymmetric" characteristics of this situation requires mortgage bankers to develop a hedging strategy that fits this unique situation. The optimal hedging strategy requires " a hedging instrument which provides the same type of asymmetric protection."<sup>24</sup> This means that a mortgage banker should employ a strategy that puts them in an equivalent position to that of the purchaser of the forward commitment. Since "the standby commitment mortgage bankers make to builders and others is equivalent to a put option contract"<sup>25</sup> (builders have the option, but not the obligation to receive mortgages at a predetermined price) mortgage bankers should use a put option on a security which has the highest correlation with the mortgages committed.

GNMA futures options or just plain GNMA options, for which there are currently no publicly active markets, would provide an excellent tool for hedging forward mortgage commitments. In fact, some mortgage bankers employ

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<sup>23</sup>Interview with Cristina I. Ray, A. G. Becker Paribas, New York, NY, March 9, 1984.

<sup>24</sup>Chiang Lui, A New Efficient Approach to Hedge Standby Commitments For Mortgage Bankers (Chicago: Kidder, Peabody & Co., [1983]), p.1.

<sup>25</sup>Ibid.

investment bankers to hedge these commitments.<sup>26</sup> Typically the investment banker writes the option on the underlying GNMA security for a premium with a stated maturity date.<sup>27</sup> Since there is no active public market in GNMA security options at this time, and because mortgage bankers and investment bankers generally keep quiet about such hedging programs,<sup>28</sup> T-bond options will be employed as the next closest, appropriate security for hedging the asymmetric risk of forward mortgage commitments.

#### Forward Commitments

Forward commitments are employed when the borrower is seeking permanent financing at a guaranteed rate in the future. Forward commitments represent a form of insurance for developers and others. Along with the many risks faced by developers, interest rate risk heads the list as the most troubling.<sup>29</sup> For example, a developer who does not engage a lender for forward mortgage commitments on a project before completion or sale leaves open the possibility that the

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<sup>26</sup>Cristina I. Ray, Interview.

<sup>27</sup>Ibid.

<sup>28</sup>Those in charge of hedging operations, who would discuss anything at all, stated that they could not disclose information about their hedging program. Interview with Paul Bevers, General Electric Mortgage, Portland, OR, February 16, 1984. Interview with Jay Maddocks, Weyerhaeuser Mortgage Corp., Los Angeles, CA, February 28, 1984.

<sup>29</sup>Interview with Ronald W. Williamson, Stonehaven Corporation, San Diego, CA.

houses built will not be sold during a given or expected time period because mortgage rates may increase. That is, if mortgage rates increase in the future, the builder will have a harder time attaining affordable financing for buyers. Hence, to limit the contingent risk that affordable financing will not be available, developers pay a commitment fee to mortgage bankers for a guarantee that financing will be granted at the commitment contract's specified rate and volume.<sup>30</sup>

In the past, permanent lenders who offered forward commitments, even though they received a fee and are legally bound to deliver under the commitment contract, often were (some still are) unwilling to fulfill their obligations. In such cases, the lender searched for "technical violations of contingencies" to avoid legal repercussions.<sup>31</sup> These situations arose primarily because lenders did not hedge their contingent liabilities, or their hedging strategy was not effective.

#### FNMA Commitments

The Federal National Mortgage Association offers long-term standby commitments to mortgage bankers and others. The commitments are made available for a fee at auctions held "bi-weekly for four-month optional delivery

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<sup>30</sup>Ibid.

<sup>31</sup>Brueggeman and Stone, pp. 438-439.

commitments to purchase FHA/VA fixed rate, FHA/VA graduated payment, and conventional fixed-rate mortgages."<sup>32</sup> Bids are made on a competitive basis by specifying a yield and an amount of principal. As of December, 1983, the fees in the auction were 1/50 of 1 percent for each bid and 2 percent for each accepted bid for a twelve month term.<sup>33</sup>

Mortgage bankers who need to hedge fixed rate loan commitments to builders have utilized FNMA standby commitments in the past. However, FNMA has become increasingly unpredictable in its acceptance of bids, and in quite a few auctions, accepted no bids at all.<sup>34</sup> Hence, mortgage bankers and other mortgage lenders must use other means to satisfy their hedging requirements. However, the accepted bids for standby commitments by the FNMA can be used as a comparison between the cost of buying direct FNMA commitments and purchasing other options, i.e., GNMA or T-bond put options.<sup>35</sup>

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<sup>32</sup>Dennis R. Capozza and George W. Gau, "Interest Rate Options," Mortgage Banking, (Special Issue, 1983), p. 64.

<sup>33</sup>Ibid.

<sup>34</sup>Ibid.

<sup>35</sup>Ibid.

## CHAPTER II

### INTEREST RATE OPTIONS

The new market in exchange traded interest rate options has provided fixed-income security investors and others with similar opportunities, with regard to patterns of risk and return, as those offered to investors in stocks and stock options. Interest rate options are based on the same general principles and serve many of the same purposes as stock options.<sup>1</sup> Debt options are traded primarily on the Chicago Board Options Exchange;<sup>2</sup> however, debt options per se are not entirely new to the securities market.

As described in Chapter II, FNMA has sold forward commitments to mortgage bankers for many years, which, for all practical purposes, are put options.<sup>3</sup> Also, many bond issues in both the corporate and municipal markets have had either puts or warrants attached to them. A put on a bond usually entitles the holder to redeem it before the stated

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<sup>1</sup>Listed Options On Debt Instruments, Jointly prepared by: American Stock Exchange, Inc., Chicago Board Options Exchange, Incorporated, and, The Options Clearing Corporation (October 1982), p. 1.

<sup>2</sup>Understanding Treasury Bond Options, Chicago: The Chicago Board Options Exchange (1982), p. 1.

<sup>3</sup>Jerry Hartzog, "Options: A New Tool For Managing Risk," Savings & Loan News, November 1982, p. 57.

maturity date. The warrants or calls on bonds allow the holder to purchase additional bonds at a given price for a period of time. However, these puts and calls are not "detachable" and can not be traded in a secondary market.<sup>4</sup> Hence, the need for exchange traded interest rate options.

On October 1, 1982, trading began on the first type of debt option: U. S. Treasury Bond Futures Options.<sup>5</sup> While T-bond futures options and T-bond options are similar in many respects, this paper focuses on the more straightforward T-bond option contract. T-bond options provide financial managers with a valuable tool for increasing their flexibility in managing interest rate risk, i.e., they provide attractive hedging opportunities against adverse interest rate movements.

Debt options of various types are currently employed by financial intermediaries, investment bankers, pension fund managers, insurance companies, corporations, and individuals with assets or liabilities that are interest sensitive. Currently there are five types of exchange traded debt option contracts: T-bond, T-bond futures, T-notes, T-bills, and Bank Certificates of Deposit (CD's).<sup>6</sup>

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<sup>4</sup>Robert W. Kopprasch, Exchange Traded Options On Fixed-Income Securities, (New York: Salomon Brothers, Inc., [1982]), p. 1.

<sup>5</sup>Hartzog, p. 56.

<sup>6</sup>Interest Rate Options, Los Angeles: The American Stock Exchange, Inc., [1982], and Mark Pitts, Options On Futures On Fixed-Income Securities, (New York: Salomon Brothers, Inc., [1982]).

Pending regulatory approval, options on GNMA securities and futures will become exchange traded.<sup>7</sup> With the advent of so many types of debt options contracts, new strategies have opened up. This chapter will present a general overview of the principles behind debt options, examine methods used to value debt options, and introduce the T-Bond option contract.

There are four standard option positions an investor can take: a long put, a short put, a long call, and a short call. Long put purchases involve the strategies of either hedging an interest sensitive asset, or speculating on the expectation of decreased security prices. A put option contract gives the holder the right, but not an obligation, to sell the underlying security at the exercise price anytime prior to the contract's expiration date.<sup>8</sup>

The "writer" of a put option (the short put position) is obligated to purchase the underlying security at the exercise price, upon execution by the holder, before the option's expiration. The writer of a put receives a premium from the buyer of the long put for incurring the risk of possible decreased future security prices.

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<sup>7</sup>GNMA Put and Call Options, Chicago: The Options Clearing Corporation [1981], p. 6. As indicated in this prospectus supplement GNMA options are currently tied up in litigation. As of March 9, 1984, GNMA options were not available in an exchange traded market. However, they are traded over-the-counter as expressed by: Ray, interview.

<sup>8</sup>Gary L. Gasineau, The Stock Options Manual (New York: McGraw-Hill Book Company, 1975), pp. 49-55.

The long call position entitles the investor, without obligation, to purchase the underlying security at the exercise price at any time prior to the option's expiration date. The purchase of call options is usually made on the expectation that the underlying security's price will increase providing a profit to the long call holder. Also, long call positions are used to protect variable liabilities from increased prices.<sup>9</sup>

Call writing, the short call position, commits the seller of a call option to deliver the underlying security at the call option holder's prerogative. Reasons for writing call options include: they provide a hedge against declining security prices; and, they also provide a hedge against increasing security prices to the extent that the premium received covers the expense of delivering an "in the money call option."<sup>10</sup>

In summary, the buyers of options are granted the right to buy or sell a security, while writers of options are committed to meet contract terms at the option of the buyer. The cost incurred by the purchasers of options is the premium received by the writer. Premiums are charged based on the writer's estimate of expected future fluctuations in the price of the underlying security.<sup>11</sup>

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<sup>9</sup>Understanding Treasury Bond Options, p. 16.

<sup>10</sup>GNMA Put And Call Options, pp. 15-16.

<sup>11</sup>Mark Pitts, The Valuation of Options On Fixed-Income Securities, (New York: Salomon Brothers, Inc., [1982]), p. 18.



Exhibit 2-A summarizes the terms and concepts used in this section.

The following is an example of a long call position: On February 16, 19XX, an investor buys a call option to purchase a 12 percent, 30-year, U. S. Treasury bond for a premium of 1.14 with an expiration month of June, and an exercise price of 100. The premium is expressed as 1 and 14/32nds, which amounts to \$1,437.50, calculated as follows:

$$\begin{aligned} \text{Cost} &= \text{Principal Value} \times \text{Premium Percentage} \\ &= \$100,000 \times .014375 = \$1,437.50. \end{aligned}$$

It must be noted that many option investors do not enter the market to buy or sell the underlying security, they do so on the expectation to buy or sell an "in the money" option in the secondary options market.<sup>12</sup>

The investor in the call option would exercise it (or sell it to a third party) if interest rates declined and raised T-bond prices by more than the exercise price plus the option's premium. Provided that T-bond prices increased by more than \$1,437.50 (on Feb. 16, 19XX, T-bond price = 100) an opportunity to purchase the T-bond at a lower than market price would exist for the investor. If interest rates increased, bond prices would fall, thereby making the call option essentially worthless beside its time value. The long call investor would allow the option to expire,

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<sup>12</sup>Capozza, p. 66.

EXHIBIT 2-ASummary of Terms and Concepts

--The Underlying Security. This is the security that may be purchased (in the case of a call) or sold (in the case of a put) by the option holder until the expiration date. Some option contracts (e.g., Ginnie Mae's) allow delivery of many different securities.

--The Strike or Exercise Price. This is the price governing the transaction that takes place when an option is exercised. For an option with multiple deliverable securities, the strike price will normally be adjusted according to the particular security delivered.

--The Expiration Date. As mentioned earlier, options can usually be exercised up to and including the expiration date, after which they will expire and become worthless.

--The Premium. This is the price paid for the option.

--The Size of the Contract. This is the amount (usually in par amount) that is covered by one contract.

--Interest Rate Option Premium Quotations. This is the price quoted for the purchase of an option. The minimum price change equals  $1/32$ ; where  $1/32 = \$31.25$  for a T-Bond with a principal value of \$100,000.00.

Source: Robert W. Kopprasch, Exchange Traded Options on Fixed-Income Securities (New York: Salomon Brothers, Inc., [1982]), p. 2.

while rates are down, and take a 100 percent loss of \$1,437.50 on all call options bought.

A long put protects an investor from rising interest rates.<sup>14</sup> To illustrate, consider the case where an investor purchases a long put on a T-bond with an expiration period of ninety days for a premium of 1.18 and an exercise price of 104. Also, assume that current T-bond prices are at 104. Consider further that interest rates increased over eighty days of the option's life, and that the T-bond price decreased to 100. In this case, the investor would exercise the option and collect a profit. The transaction may occur thusly: investor purchases the T-bond with the same coupon rate and expiration date in the market for \$100,000.00, and then exercises his put option right to sell the security for \$104,000.00. The investor's net profit would equal:

$$104,000 - 100,000 - 1,562.50 = \$2,437.50 \quad (1,562.5 = \text{premium} = .015625 \times 100,000.)$$

In percentage terms, the put buyer has received a return of 156 percent in under three months time on an investment of \$1,562.50. On the other hand, if T-bond yields fell or remained constant over the option's life, the buyer would lose \$1,562.50-- the T-bond premium. However, most put option buyers who use them for hedging purposes, consider this loss as the cost of insurance for \$100,000.00 of T-bonds for a ninety-day period.<sup>15</sup>

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<sup>14</sup>Understanding Treasury Bond Options, p. 18.

<sup>15</sup>John W. Labuszewoski and James F. Meisner, "Controlling Mortgage Pipeline Risk," Mortgage Banking, (November 1983), p. 50.

## EXHIBIT 2-B

THE RELATIONSHIP BETWEEN INTEREST RATE CHANGES (YIELDS)PRICES, AND THE VALUE OF DEBT OPTIONS

Interest rates (Yields) ↓ = price ↑ = Call Value ↑  
 Put Value ↓

Interest rates (Yields) ↑ = price ↓ = Call Value ↓  
 Put Value ↑

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Source: Listed Options on Debt Instruments, p. 5.

With the relationships represented in Exhibit 2-B in mind, the various strategy positions can be examined graphically. The option positions to be in for speculating on or hedging against an interest rate increase would be either a put or a short call. The basic strategies to employ with expectations of an interest rate decrease would either be the long call position or the short put position.<sup>15</sup>

Exhibit 2-C illustrates the return pattern associated with a call option on a 12 percent, 20-year bond.<sup>16</sup> The call's strike price equals 100 and its expiration date is three months. At all prices below 100, the call's intrinsic value is zero because the bond could be purchased for less in the market without exercising the option. The call options's intrinsic value, for prices above 100, equals the difference between market price and exercise price (e.g., exercise price = 100, market price = 104; intrinsic value = 4.)<sup>17</sup>

Exhibit 2-C also represents the profit pattern for an investor who paid a premium of 3 points. The maximum the investor can lose from his investment is the premium, no matter how much the bond price falls. Also notice that the premium paid determines the call investment's breakeven point. At all bond prices above the break even bond price,

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<sup>15</sup>Ibid., p.4.

<sup>16</sup>Kopprach, pp. 3-6.

<sup>17</sup>Ibid.

the investor receives a profit upon exercise.

Exhibit 2-D translates the profit pattern in Exhibit 2-C into a yield framework that illustrates the returns of the call option versus the yield of the deliverable bond at expiration. The breakeven bond price in exhibit 2-C translates into a breakeven yield in Exhibit 2-D. It must be noted that the diagonal line in Exhibit 2-D is an estimation--the real curve is concave. As illustrated in these two exhibits, the breakeven bond yield is a function of the call's premium, and the slope of the price yield curve, where the slope of the price yield curve of a bond is a function of maturity, yield, coupon rate, and the number of coupon payments per year.<sup>18</sup>

As described above, a put option is an option held by the buyer to sell a security at a specified price. The intrinsic value of a put, as shown in Exhibit 2-E, is the difference between the strike price and the market price, when the market price at expiration is less than the strike price (e.g., strike price = 100, market price = 95; intrinsic value = 5).<sup>19</sup> If the market price is above or equal to the strike price at expiration, then the intrinsic value equals zero. The intrinsic value and profit pattern of a put option on a 12 percent, 20-year bond is illustrated in Exhibit 2-E.<sup>20</sup> The put's expiration period is ninety

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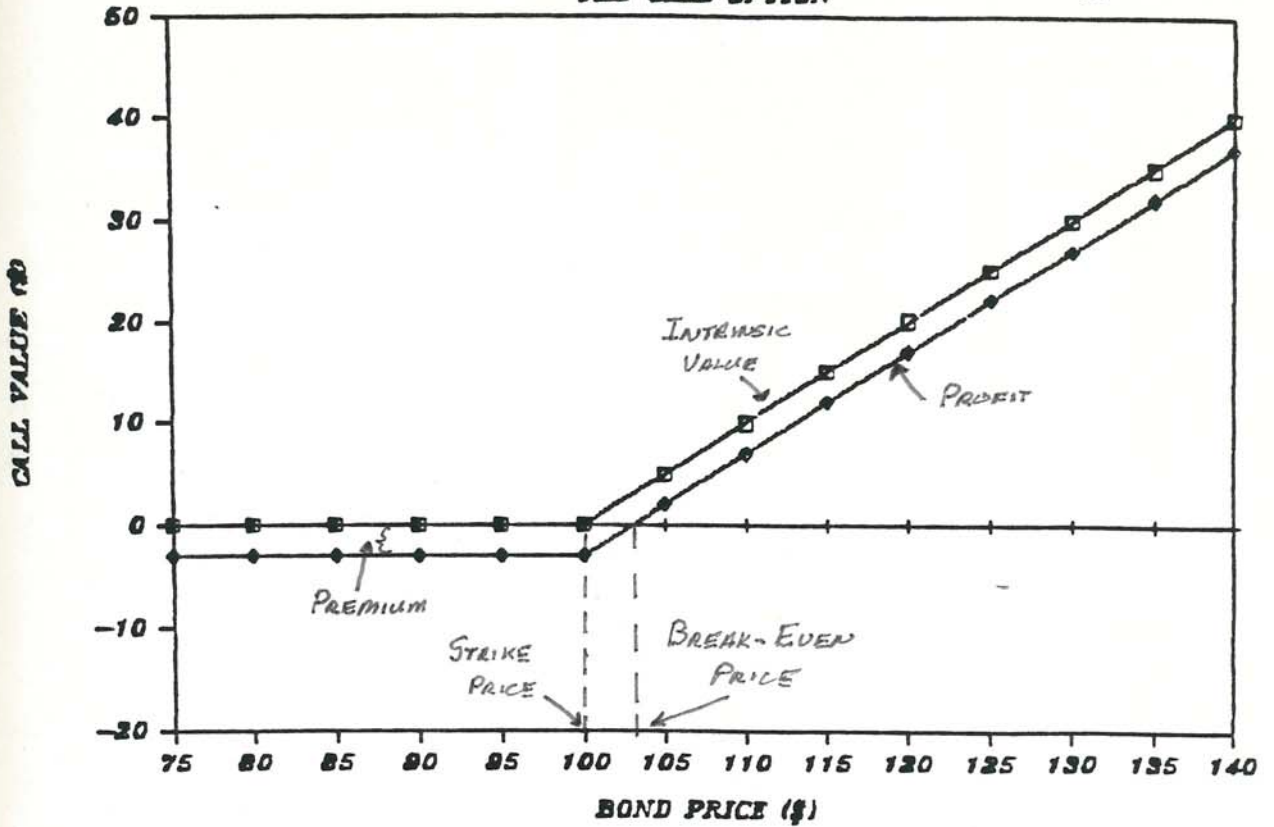
<sup>18</sup>Ibid.

<sup>19</sup>Ibid., pp. 8-4.

<sup>20</sup>Ibid.

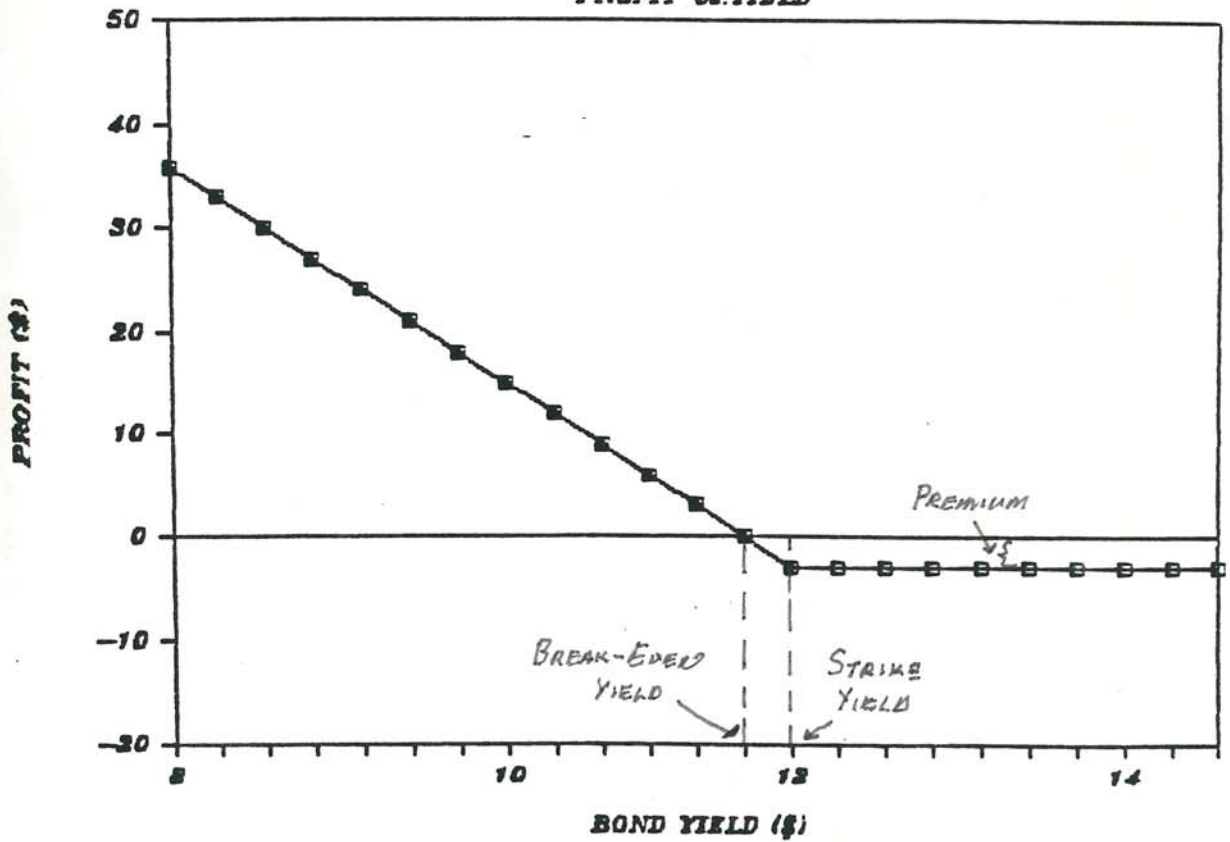
# EXHIBIT 2-C

THE CALL OPTION



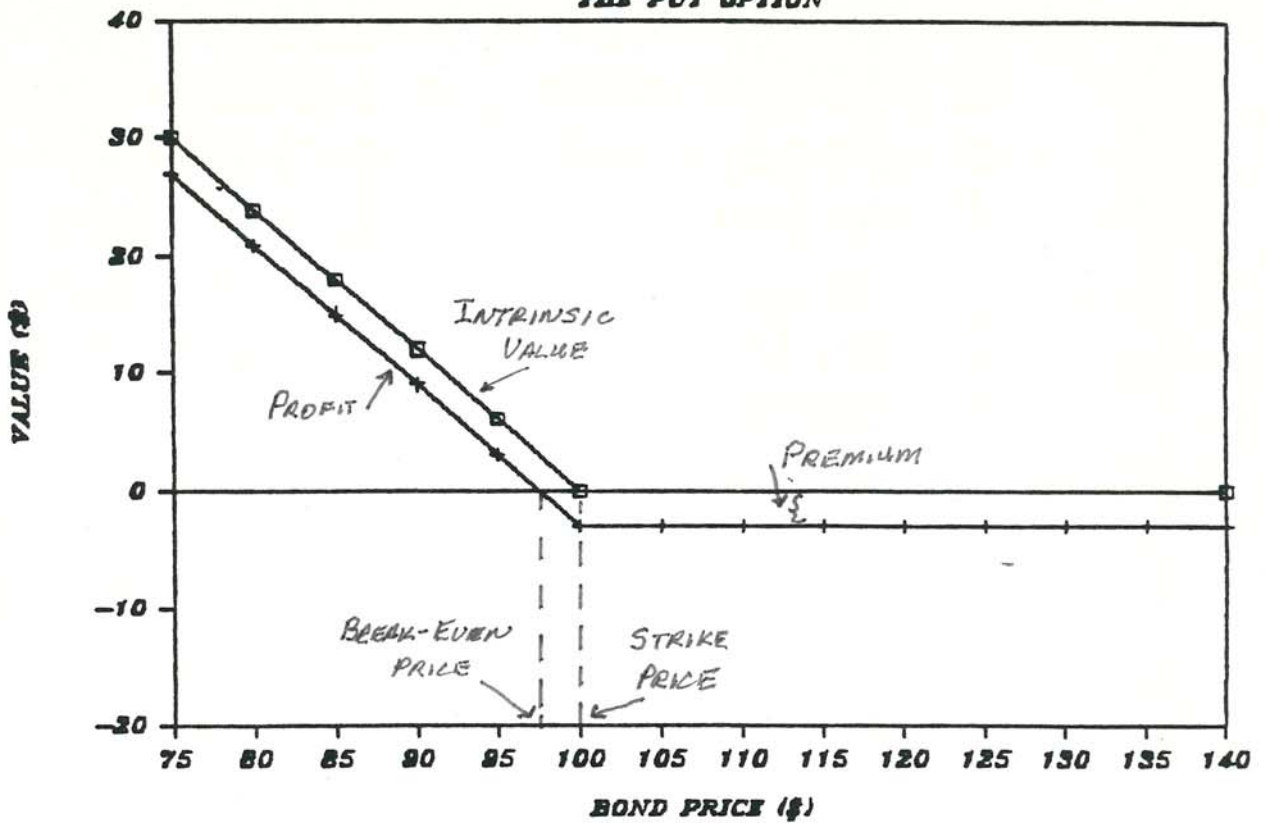
# EXHIBIT 2-D

PROFIT vs. YIELD



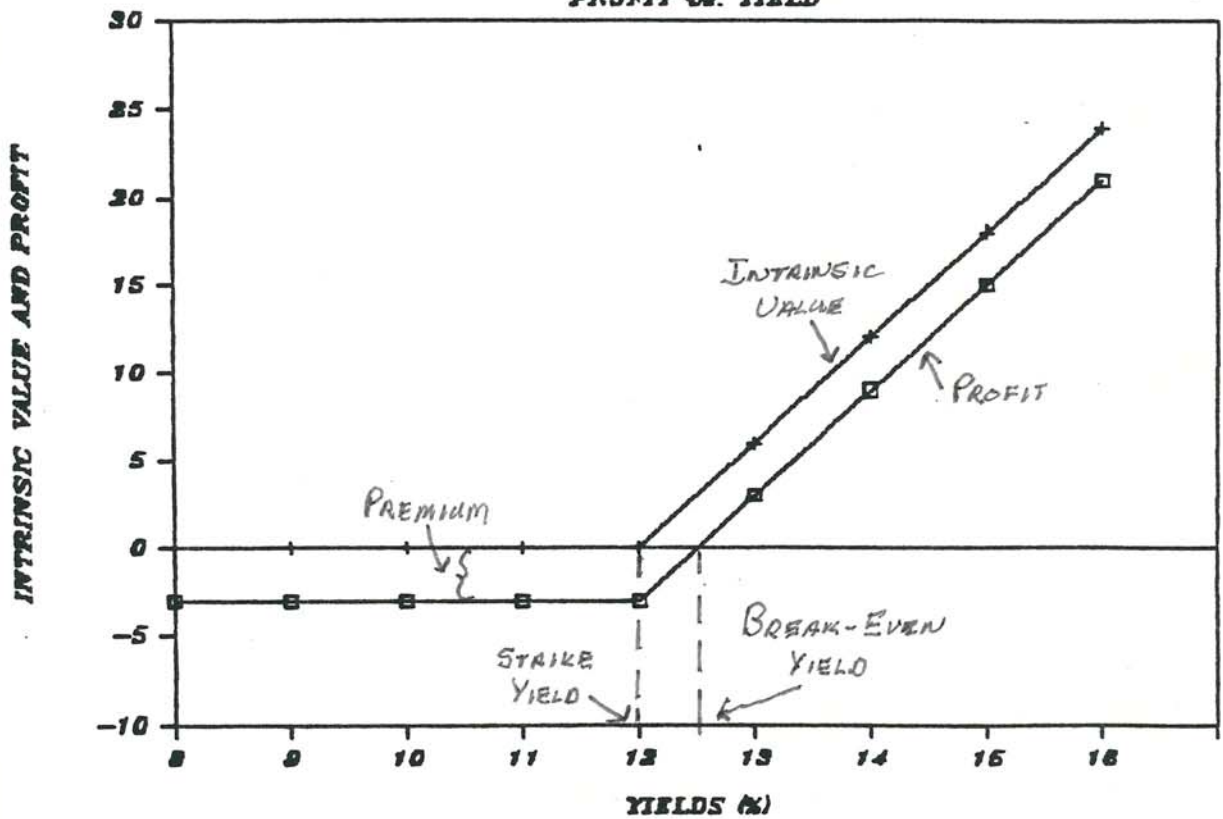
# EXHIBIT 2-E

## THE PUT OPTION



# EXHIBIT 2-F

## PROFIT vs. YIELD





days and has an exercise price of 100.

The profit pattern and breakeven point is analogous to that of the call's in Exhibit 2-C. Here too, the investor paid a premium of 3 points which determines the profit or loss line in Exhibit 2-E.

Exhibit 2-F is analogous to Exhibit 2-D; it illustrates the profit pattern corresponding to the various yields at expiration for the bond. Here again, the yield-profit curve is approximated -- it should be concave.

Exhibits 2-C through 2-F also represent the return to call and put writers. In Exhibit 2-D, for example, the premium is the put writer's return or profit up to the breakeven point, while prices below this point represent losses.

The graphs in Exhibits 2-C through 2-F approximate the return patterns for speculative positions, but do not represent the return schedules for a given hedged position.<sup>21</sup> The hedging examples in Chapter III will examine these possible returns. However, the general effects of the different options positions can be defined: A long put increases the lowest possible return on a hedged position; a short call reduces the highest possible return on a hedging strategy; a long call increases the maximum return on a hedged portfolio; and, a short put decreases the lowest possible return on a hedged position.<sup>22</sup>

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<sup>21</sup>Hartzog, p. 59.

<sup>22</sup>Ibid., pp. 59-60.

Appendix D lists the major differences between option and futures under the categories of: major stipulations, costs, key determinant of cost, delivery date, and possible positions. The other possible positions obtainable under debt option strategy include: spreads, straddles, and combinations. However, here just a brief description of them will be made because there are many variations of each different type, and because of their complexity.<sup>23</sup> These positions either elevate the degree of leverage in a position or manipulate the degree of risk in a hedging strategy. A spread position is where an investor is long one or more options on a security and short one or more options on the same security, but of a different series; i.e., a different strike price or expiration date. The straddle position is described by being simultaneously long in both put and call options of equal or unequal amounts of the same strike price and expiration date. A combination position is similar to a straddle except different strike prices or expiration dates or both are employed in either the put or call option contracts.<sup>24</sup>

Appendix E lists equivalent positions with regard to holding a security and the side of an option contract an investor may take. The list singles out positions that

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<sup>23</sup>A few of these variations are described in detail in: Max G. Ansbacher, The New Options Market, 2 ed. (New York: Walker and Company, 1975), Chapter 6.

<sup>24</sup>Ibid.

should be avoided because of their equivalency. That is, it is cheaper to just enter into the end position.

### The T-Bond Option Contract

The primary uses of T-bond options are: if an investor has a portfolio of U.S. Treasury bonds, or other debt instruments, then T-bond options can be used as a means of protecting their position against adverse interest rate movements; or, if an investor has an expectation about the future interest rate yield curve, then T-bond options can be used for speculation and profit.<sup>25</sup> To fully understand the T-bond options contract one must be familiar with the nature of the underlying securities market.<sup>26</sup> The underlying security of a given T-Bond option contract is the actual T-bond with the stated maturity and coupon rate in the contract. Since the price of the underlying security is often the most important determinant affecting the price of an option, one should be aware that the price of a fixed income security is inversely related to the direction of change in required yield. Graph A in Appendix I illustrates this relationship.

### Contract Terms

#### Unit of Trading

T-Bond option contracts cover two principal amounts, \$100,000 for a "standard" contract, and \$20,000 for a

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<sup>25</sup>Understanding Treasury Bond Options, pp. 3-4.

<sup>26</sup>Ibid., pp. 5-10.

"mini-series" contract.<sup>27</sup>

#### Expiration Dates

T-Bond options are traded with three expiration months at one time, from the following months: March, June, September, and December. For example, in February 1984, options on T-bonds were listed with expiration months of March '84, June '84, and September '84. T-bond options expire on "the Saturday after the third Friday of the expiration month."<sup>28</sup>

#### Exercise Prices

The CBOE introduces exercise prices that bracket the current market price of the underlying T-bond in intervals of 2 points. Other strike prices are listed in accordance with the price volatility of the market. For example, if the current price of an underlying T-bond were 98, there would at least be strike prices available of 96, 98, and 100 and possibly ones of 102 and 104 as well.<sup>29</sup>

#### Premium Quotations

The chart below illustrates the premium quotations for both contract sizes<sup>30</sup>:

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<sup>27</sup>Ibid., p. 10.

<sup>28</sup>Ibid., p. 11.

<sup>29</sup>Ibid.

<sup>30</sup>Ibid., p. 12.

## Exhibit 2-G

	\$100,000 T-Bond	\$20,000 T-Bond
1 point	\$ 1,000.00	\$ 200.00
1/32nd	31.25	6.25

To determine the price from a quote of 2.07 (i.e., 2-7/32 points) on a T-bond option contract with \$100,000 principal value, one simply multiplies 2.22 percent by 100,000, which equals \$2,218.75.

Debt Option Valuation

An option's price is determined, theoretically, by the current price of the underlying security, the exercise price, the price volatility of the underlying security, the current risk-free rate of return, and the time to maturity of the option.<sup>31</sup> The premium paid for an option reflects both the risk exposure of both sides of a transaction and the level of interest rates. In general, an option's price has two elements: the current intrinsic value (as described above) and the commitment fee or time value. An option is said to be "in the money" if the intrinsic value of the option is greater than the premium or cost of the option, "out of the money" if the current market value is negative, and "on the money" if the current market value is zero.<sup>32</sup>

The time value component of an option's price takes

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<sup>31</sup>Pitts, p. 18.

<sup>32</sup>Kopprasch, Exchange Traded Options with Fixed Income Securities, p. 10.

into consideration the risk of possible adverse interest rate fluctuations over the life of an option. This portion of an option's premium,

...represents the value paid by the option holder to the option writer for assuming the obligation to buy or sell a security at a predetermined price for the duration of the option.<sup>33</sup>

Various models have been developed to determine the fair value of a given type of option or options in general,<sup>34</sup> but as of this writing, no one perfect method has been developed.<sup>35</sup> Below, an analysis using a modified version of the Black-Sholes Option Pricing Model will be employed to show the various effects on an option's price for varying certain assumptions on a T-bond option contract.

#### The Black-Sholes Model

The Black-Sholes Option Pricing Model employs a riskless hedge valuation technique to determine the price of an option.<sup>36</sup> However, it does so at a cost: simplifying

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<sup>33</sup>Hartzog, p. 53.

<sup>34</sup>A few of the models developed are discussed in: Gatineau, pp. 179-203 and another interesting model is presented in: Richard V. Rendleman and Brit J. Bartter, "The Pricing of Options on Debt Securities,": Journal of Financial and Quantitative Analysis 25 (March 1980): 11-15.

<sup>35</sup>Ray, Interview.

<sup>36</sup>Fisher Black and Myron Sholes, "The Valuation of Option Contracts and a Test of Market Efficiency," The Journal of Finance 27 (May 1972):400.

assumptions are made that do not conform to reality. The assumptions made by the Black-Sholes model are listed in Exhibit 2-H.

The following is a more general description of what these assumptions mean: The risk-free rate for borrowing and lending for the current date is known and constant over the period before expiration. Yield may increase rapidly, however, they cannot "jump" or increase drastically during any one trading day; this corresponds to the lognormal distribution for prices. It is assumed that instantaneous action will be taken in response to adverse price changes. Although taxes are always a factor, the model ignores them for simplicity. Also, transaction costs (commissions and fees) are assumed small enough to be disregarded.

The riskless option hedge technique is based on the concept of a neutral option hedge, i.e., subject to its various assumptions, a perfectly hedged position is set up, given a long position in the underlying security, and a short position in options on the security, or a long position in options, and a short position in the security.<sup>36</sup> By a perfectly hedged position, the model implies that over a given security's price interval, within a defined range of the current price, any profit made from an immediate increase in the security's price would be completely offset by a loss on the option position or vice versa. Thus, the model is based on the principle that "options can completely

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<sup>36</sup>Rendleman and Bartter, p 12.

## EXHIBIT 2-H

KEY ASSUMPTIONS OF THE BLACK-SHOLES MODEL

1. The short-term interest rate is known and is constant through time.
2. The security price follows a random walk in continuous time with a variance rate proportional to the square of the security price.
3. The distribution of possible stock prices at the end of any finite interval is lognormal.
4. The variance rate of return on the security is constant.
5. The security pays no dividend and makes no other distributions.
6. The option can only be exercised at maturity.
7. There are no commissions or other transaction costs in buying or selling the security or the option.
8. It is possible to borrow any fraction of the price of a security to buy it or hold it, at the short-term interest rate.
9. A seller who does not own a security (a short seller) will simply accept the price of the security from the buyer and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date. While this short sale is outstanding, the short seller will have use of, or interest on, the proceeds of the sale.
10. The tax rate, if any, is identical for all transactions, and all market participants.



eliminate market risk from a stock portfolio.<sup>37</sup> Since the position is deemed theoretically riskless, the option premium at which the hedge yields a return equal to the risk-free, short-term interest rate is considered the "fair" value of the option.

The Black-Sholes formula is expressed as follows:<sup>38</sup>

$$V_c = P_s N(d_1) - S e^{r(t-t^*)} N(d_2); \text{ where}$$

$$d_1 = \frac{\ln(P_s/s) + (r+1/2 v^2)(t^*-t)}{v(t^*-t)}$$

$$d_2 = \frac{\ln(P_s/s) + (r-1/2 v^2)(t^*-t)}{v(t^*-t)}$$

$V_c$  = PV of a call option

$P_s$  = current security price

$S$  = striking or exercise price

$N(d^*)$  = cumulative normal density function evaluated at  $d^*$

$r$  = risk-free interest rate

$t$  = current date

$t^*$  = maturity date of option

$v^2$  = variance rate of return on the security

$e$  = base of natural logarithm = 2.71828

$\ln$  = natural logarithm

The price of a put option is expressed in like manner except

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<sup>37</sup>Kopprasch, Exchange Traded Options with Fixed Income Securities, p. 3.

<sup>38</sup>Ibid., p. 197.

The price of a put option is expressed in like manner except the minus sign becomes positive and  $P_S$  becomes negative.<sup>39</sup>

Exhibit 2-I represents the model in a computer program written in the Interactive Financial Planning System (IFPS) language.<sup>40</sup> The program is documented to explain what each expression does. Exhibits 2-J through 2-P represent trial runs of the program given various inputted data. The data that must be included are: price (current T-bond price), exercise price, risk-free rate (T-Bill rate), time to exercise (number of days before expiration divided by 360) and the standard deviation of ROR (the standard deviation of the T-bond's rate of return). Each column of output in the exhibits represents one run of the program. The inputted variables and calculated or interpolated variables are also listed for each run. As demonstrated by the outputs in the exhibits, the present value of a put or call option can vary significantly with respect to changes in the variables inputted. Exhibits 2-Q through 2-S graph the present value of a put with respect to exercise price, time, and risk free rate. For example, when the exercise price was varied in the computer program the price or present value of the option decreased as the exercise price increased. This is clearly shown in the graph in Exhibit 2-Q. The data graphed in the exhibits 2-"Q", "R", and "S" was taken from exhibits

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<sup>39</sup>Rendleman and Bartter, p. 13.

<sup>40</sup>Paul Gray, Student Guide to IFPS (New York: McGraw-Hill Book Company, 1983), pp. 244-248.

2-"L", "N", and "M" respectively. As the data in the exhibits illustrate, increases in the price of the underlying security causes an increase in the call price and a decrease in put price. Exhibit 2-T describes the price effects on both put and call options from increases in the data inputed.